Structural Integrity Analysis

1. STRESS CONCENTRATION

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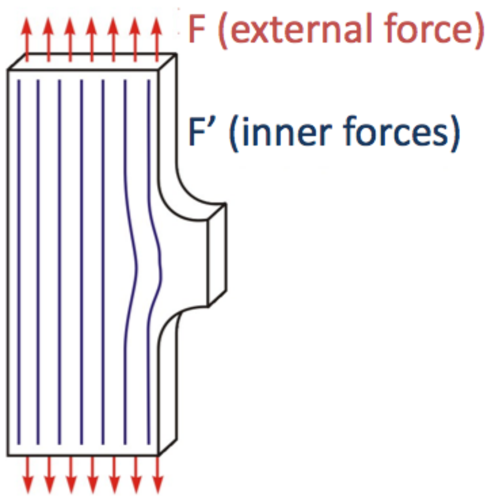
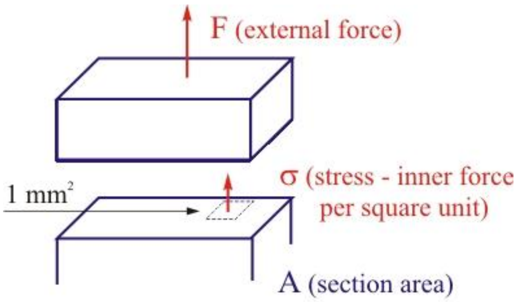
1.1 STRESSES AND CONCENTRATORS

1.1.1 Stress

An applied external force F causes inner forces in the carrying structure. Inner forces F' are shown by the blue lines spread throughout the structure. Inner forces are distributed differently in each part of the structure.

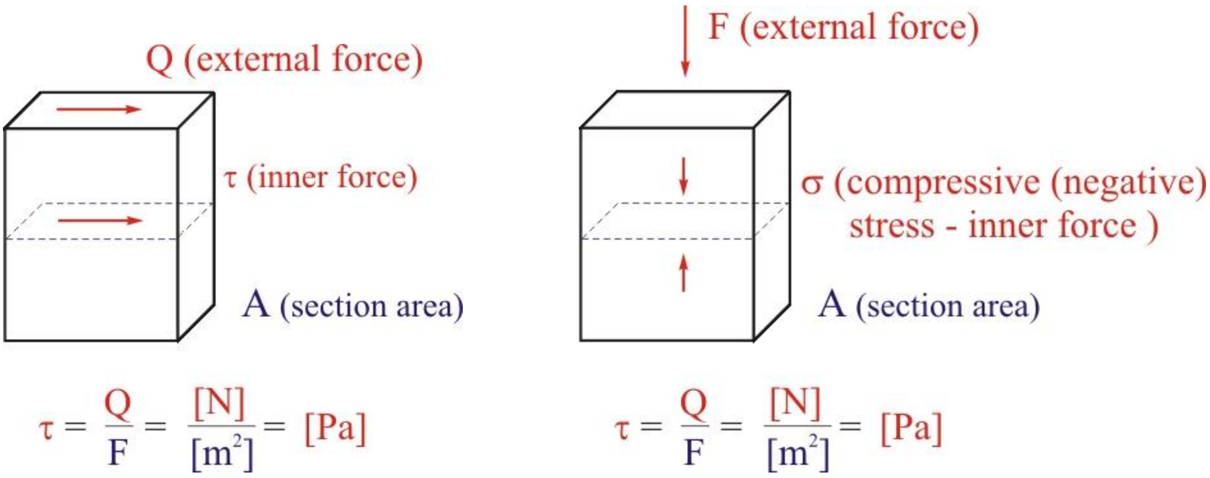


To describe the inner forces in a section of the structure we use stress defined as the force divided by the cross-sectional area. Stress corresponds to the force acting on a unit of area (square millimeter, square inch, square meter, etc.). A smaller cross-sectional area creates a larger amount of stress under the same external force.



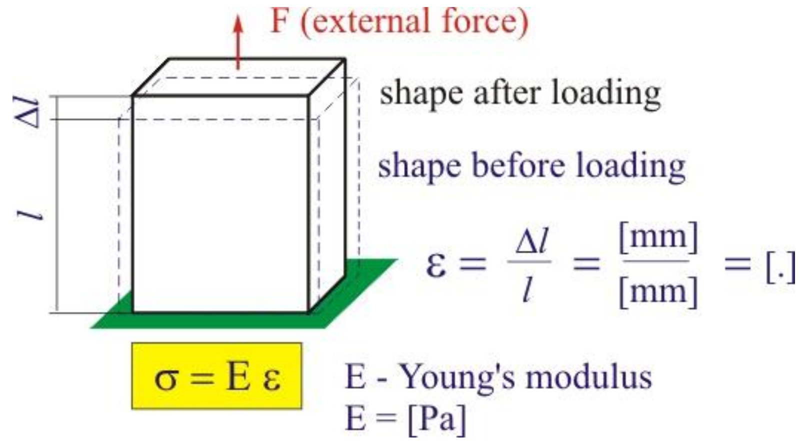
$$\sigma = \frac{F}{A} = \frac{[N]}{[m^2]} = [Pa]$$

As a result, compressive and shear forces also cause compressive and shear stresses. As a rule in complex structures, tensile stress results in compressive stress in the perpendicular direction.



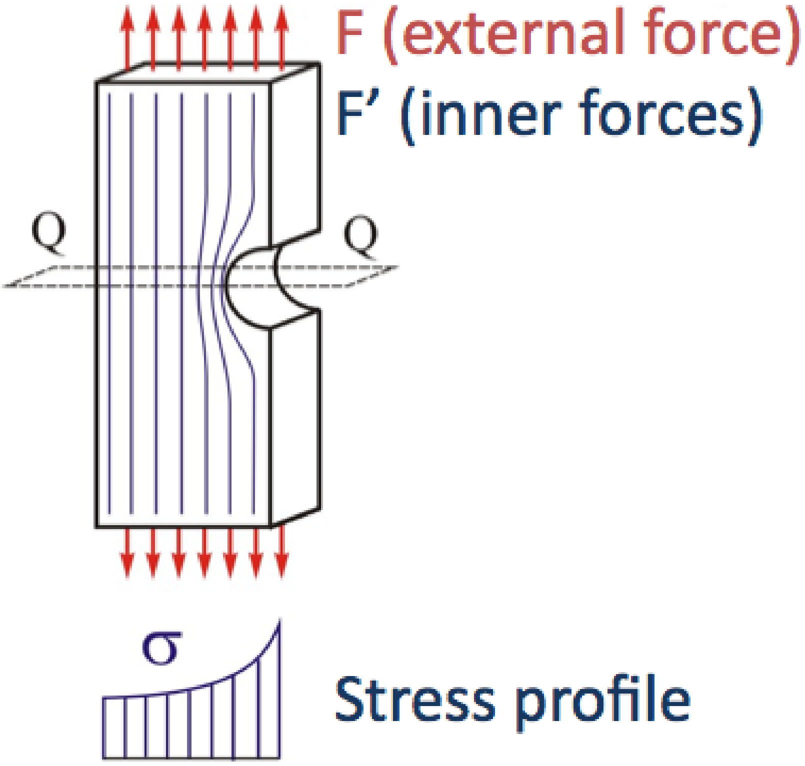
1.1.2 Strain

The shape of a structure will change under loading. For example, a structure will elongate under tension. To estimate this process the value of the strain was introduced. Strain is the ratio of elongation to initial length and is therefore dimensionless. It is also linearly proportional to stress.

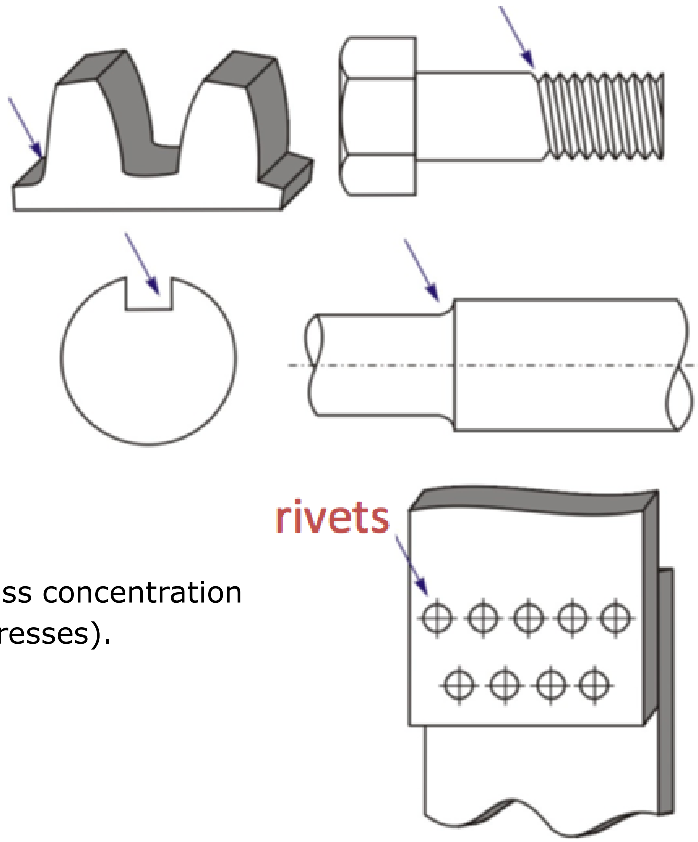


1.1.3 Concentrators

When inner forces go around holes or notches, they will concentrate near such "obstacles." Stress concentrators are areas that tend to magnify the stress level within a part. Stress that is higher in one area than it is in surrounding regions can cause the part to fail. If the radius of curvature in the notch tip is very small or if there is no radius (crack), the stress level is very high. Sharp corners are especially critical.



What can serve as stress concentrators?

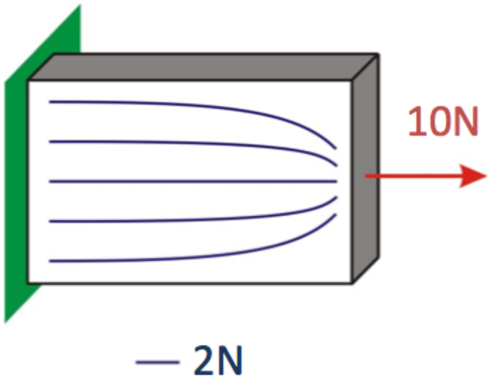


- Holes and slots
- Notches or grooves
- Ribs, gussets, and posts
- Sharp wall thickness transitions
- Surface roughness
- Bosses
- Corners

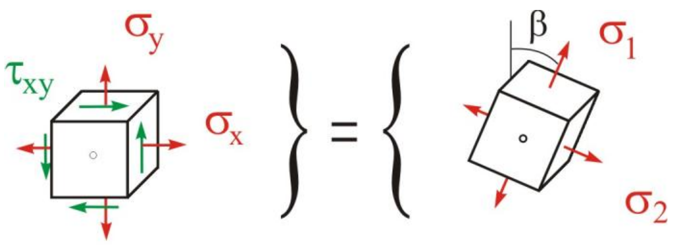
The mentioned design features will not cause stress concentration in those parts where there are no inner forces (stresses).

1.2 FORCE LINES

Force lines show forces inside a structure. They have dimensions (units) of external force. If external force 10 N is shown by five force lines, each line has its "price" that is equal to 10 / 5 = 2 N. Usually, all force lines have a constant price in one figure.

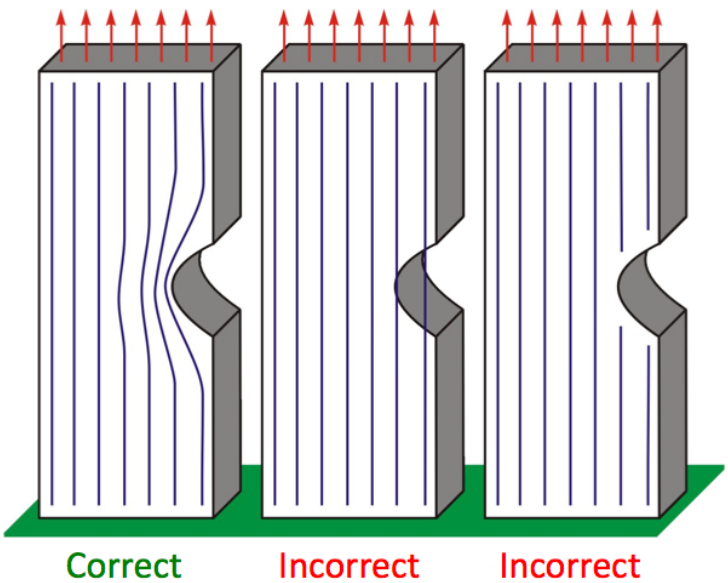


For any state of stress, it is possible to determine the direction of the maximum tensile stress. This stress is called the main stress. Force lines are drawn by the integration of the main stresses. Mathematical methods are used to draw force lines. We use a few simple rules to present force lines:

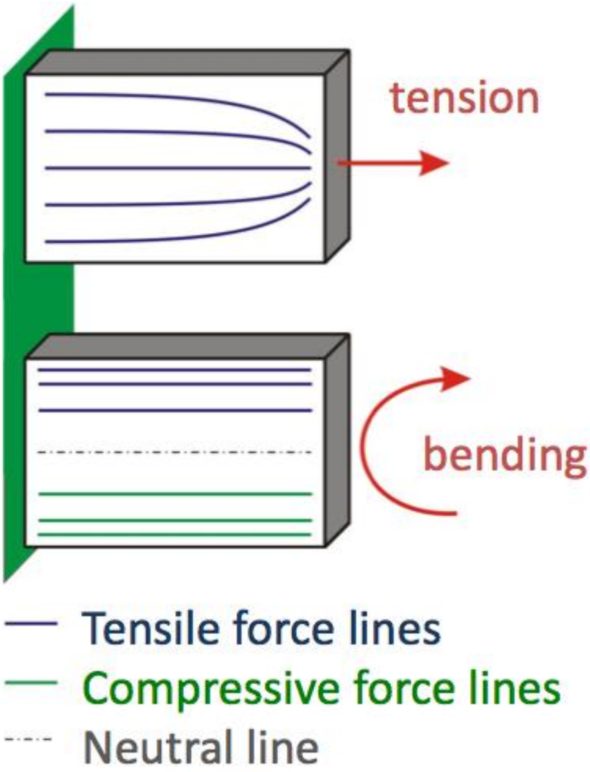


$$\sigma_1 > \sigma_y, \sigma_x, \sigma_2$$

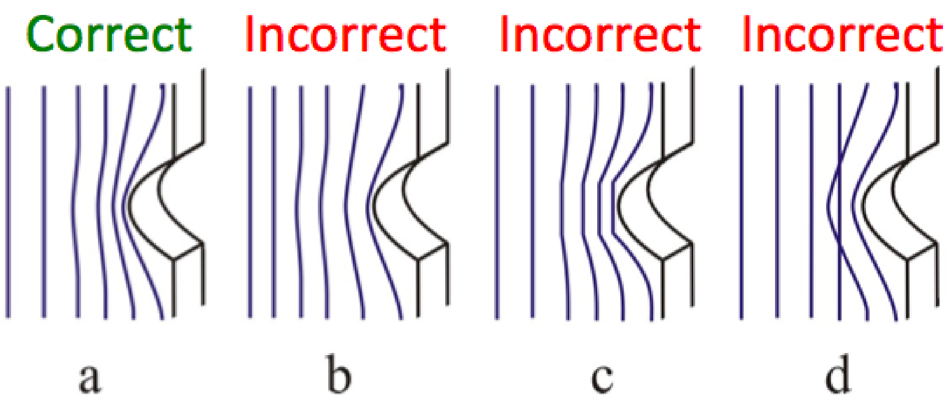
Force lines start on the surfaces where the external forces are applied. They go around "obstacles" such as holes and notches.



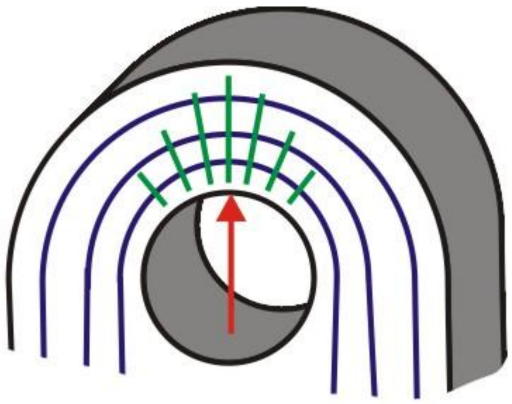
Force lines are distributed uniformly for tension. Their density is higher at the edge of a beam under bending.



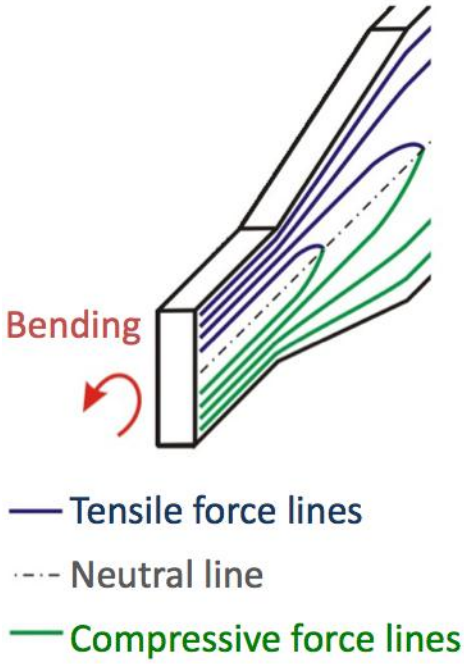
By passing around the "obstacles," the force lines concentrate in the tip (a).
They are not uniformly distributed near the concentrator (b).
There is no sudden change of direction (c).
Force lines cannot intersect each other (d).



Both tensile and compressive force lines can be used in the analysis. Usually, compressive lines are perpendicular to tensile lines.

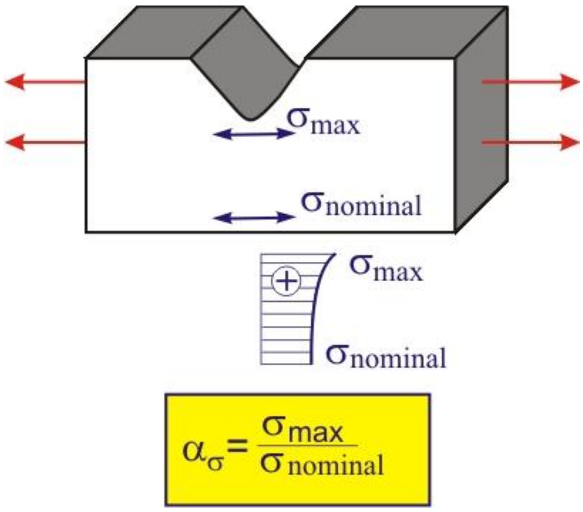


Force lines will compensate for each other under bending.

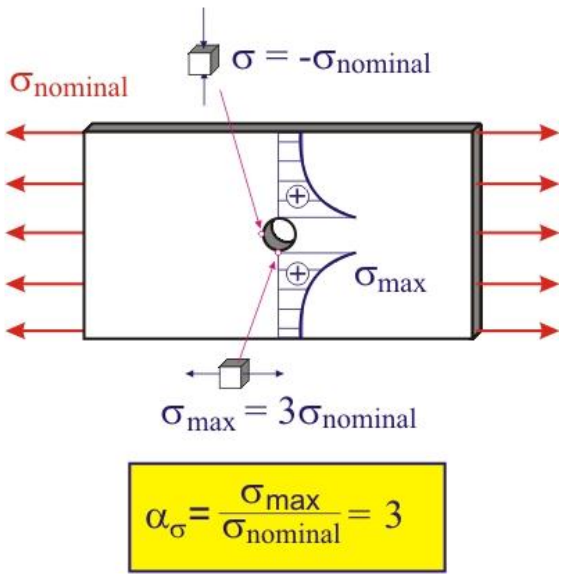


1.3 STRESS CONCENTRATION FACTOR

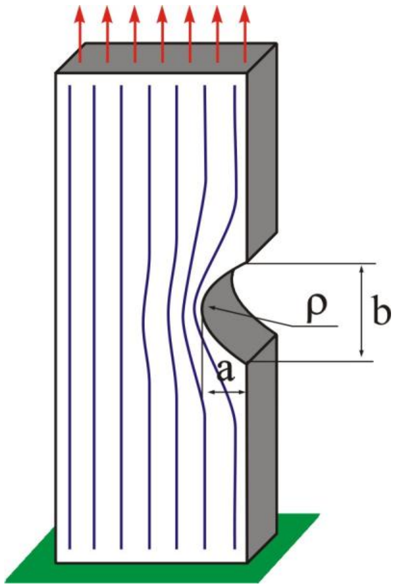
Stress concentrators cause high stresses in the structure. The stress concentration factor is the ratio of maximum stress to nominal stress. It is greater than 1 and a dimensionless parameter. There are different formulas for nominal stress, which usually occurs in the absence of concentrators.



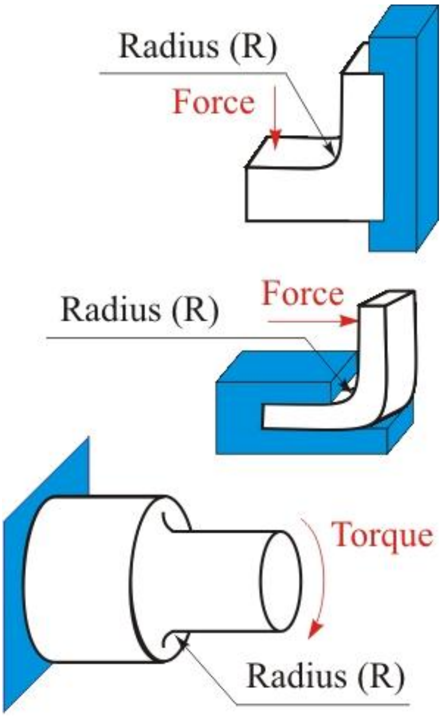
The authors of the theory of elasticity proved that tensile stress near a hole in a wide plate is three times higher than nominal stress. This means that the stress concentration factor is equal to 3 in this case.



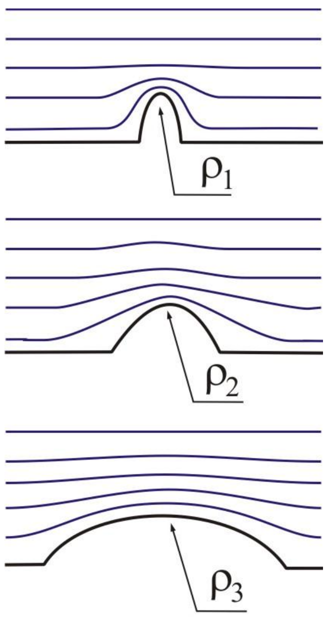
The stress concentration factor increases depending on:
a) The larger size of the obstacle on force line path a
b) The smaller size of the obstacle along force line path b
c) The smaller radius of curvature in the notch tip



Theoretically, if the radius tends towards 0 (sharp crack), the stress concentration factor tends towards infinity. That conclusion is correct only for an ideal elastic body. In real structures, the stress concentration factor is finite due to plasticity and microstructural changes. Thus, the stress concentration factor increases as the radius of curvature in the notch tip decreases.



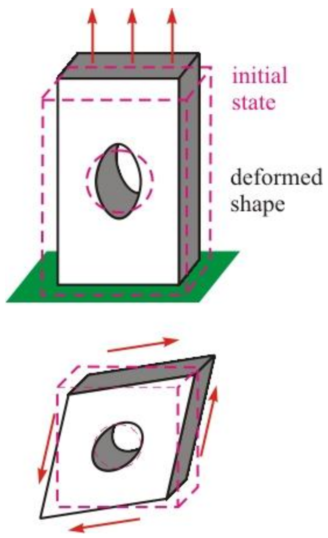
A larger radius in the notch tip will lower the stress concentration.



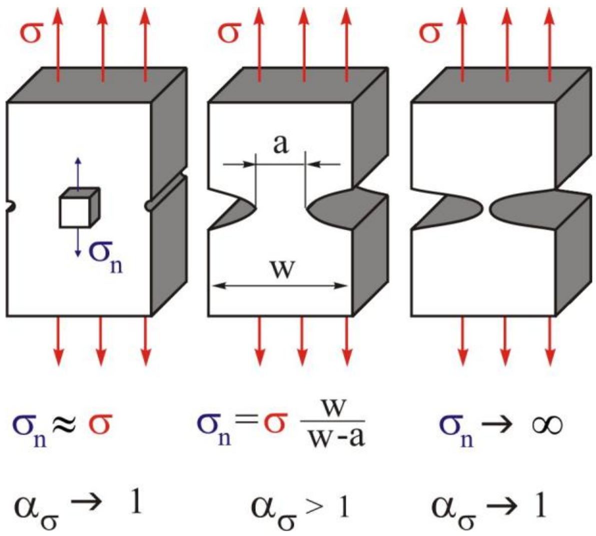
$$\rho_3 > \rho_2 > \rho_1$$
$$\alpha_\sigma^3 < \alpha_\sigma^2 < \alpha_\sigma^1$$

1.4 THEORY OF ELASTICITY AND STRESS CONCENTRATION

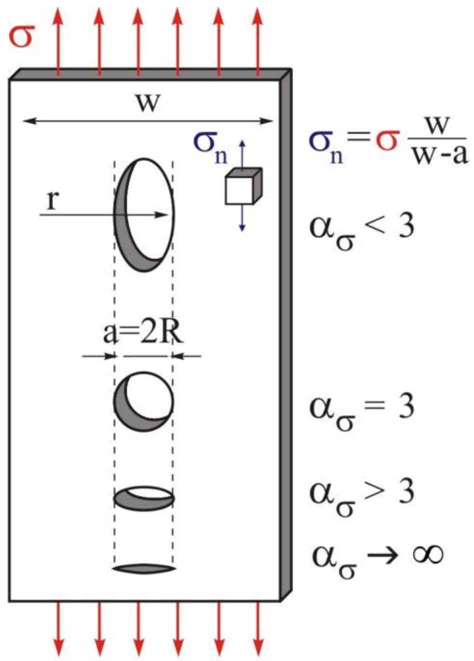
At the beginning of the 19th century, the authors of the theory of elasticity showed that an elastic body with a hole will change its form by extending in one direction and compressing in another. A round hole is converted into an elliptical one with a larger axis along the tensile direction.



Usually, nominal stress is defined as the average stress in a cross-section. Consider the three situations to the right. In the last case, the stress is very high due to the large amount of nominal stress. The stress concentration for the third scheme is lower.



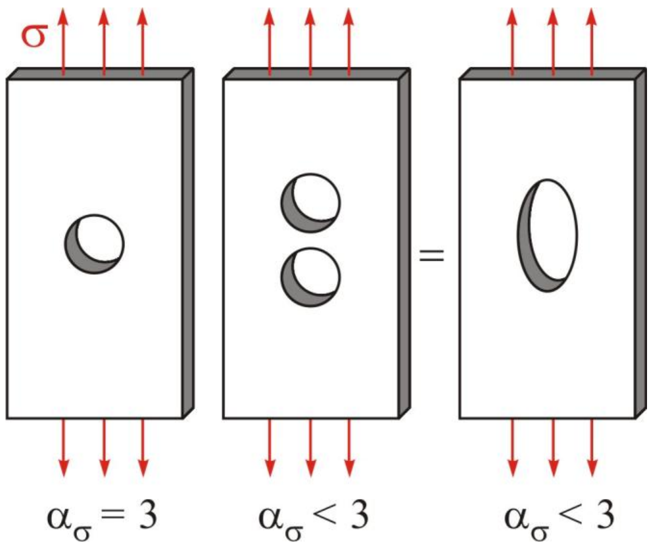
Elliptical holes can be problematic if the larger axis is perpendicular to the applied tension. In this case, the width of the "obstacles" on the force line path is large and the radius of curvature in the notch tip is small.



$$\alpha_\sigma = 1 + 2\sqrt{\frac{R}{r}}$$

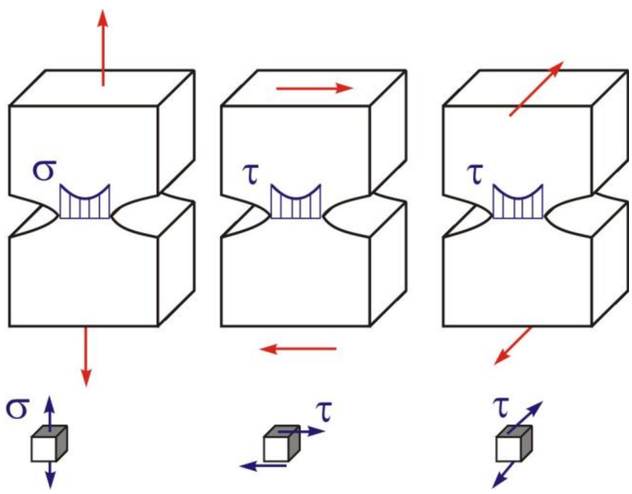
The formula shown here is valid for an elliptical hole in an infinite plate. Only in theory does the stress concentration factor tend towards infinity for a crack (radius of curvature is equal to 0).

Two holes lying along the same axis of tension have a stress concentration lower than 1, while two holes on the same "obstacle" on the force lines path act as the elliptical hole.

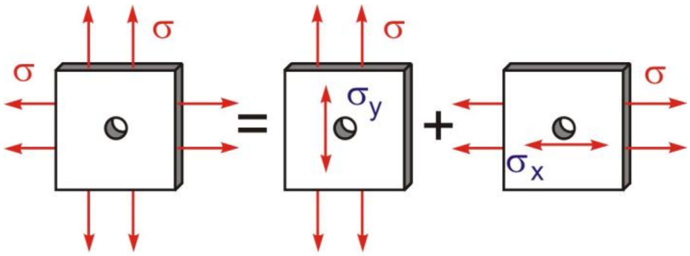


1.5 MIXED MODE STRESS CONCENTRATION

In the notch, the stress is as high for tension as it is for shear. The stress concentration factor has the same value for different loading schemes on the same geometry.



For biaxial tension or more complicated loads, the principle of superposition is applied.



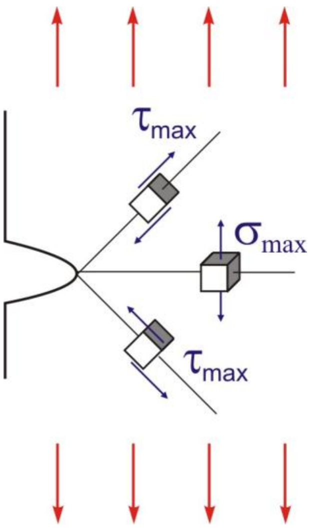
$$\sigma_y^{max} = 3\sigma + -\sigma$$

$$\sigma_x^{max} = -\sigma + 3\sigma$$

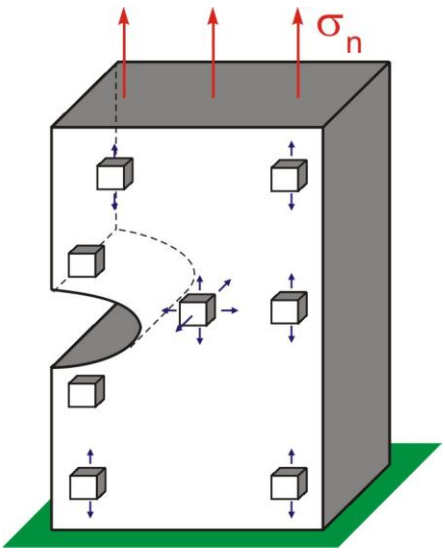
$$\sigma_y^{max} = \sigma_x^{max} = 2\sigma$$

For example, biaxial tension is the superposition of two applications of tension and this has twice the maximum stress as nominal stress.

The maximum tensile stresses act on a line perpendicular to the tension line. The maximum shear stresses act on an angle to the tension line.

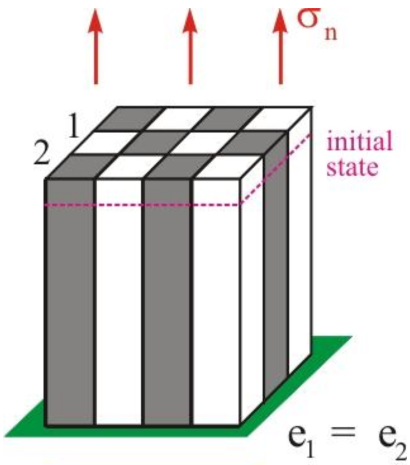


There is three-dimensional state of stress in the notch. The complex stress state is only in the small area near the tip of the notch. In the other points of the structure, there are tensile stresses only or no stresses at all ("dark corners").



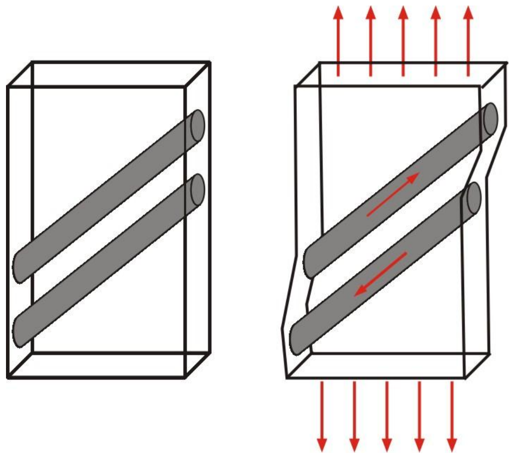
1.6 STRESS CONCENTRATION IN AN ANISOTROPIC BODY

Force lines are concentrated in the rigid components of composite materials. The strains are equal for all components, but the stress is higher in rigid components.

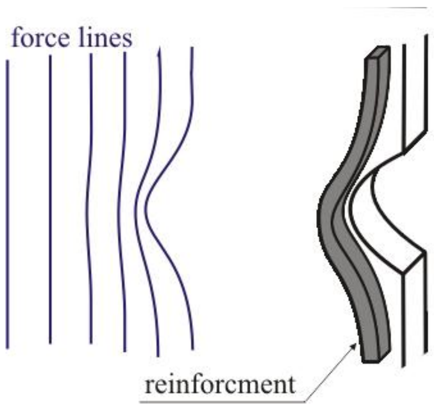


$$\sigma_1 = \sigma_2 \frac{E_1}{E_2}$$

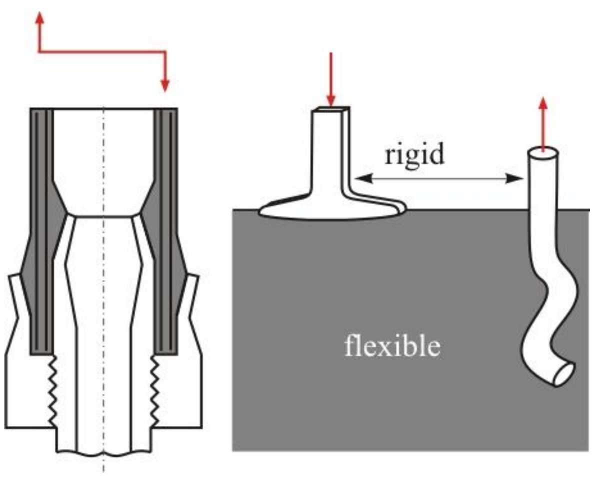
Rigid fibers define the deformed shape of composite materials. The material in a more flexible matrix has larger shear deformations.



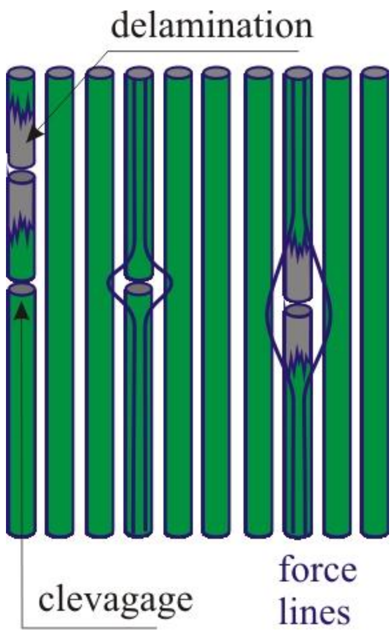
A plate is reinforced with a concentrator according to the distribution of the force line. An effective method is to place reinforcement bars along the main force line.



For proper design, the zones of transition from rigid to more flexible components should not include sharp corners, sharp bends, thin flexible layers, and so on.

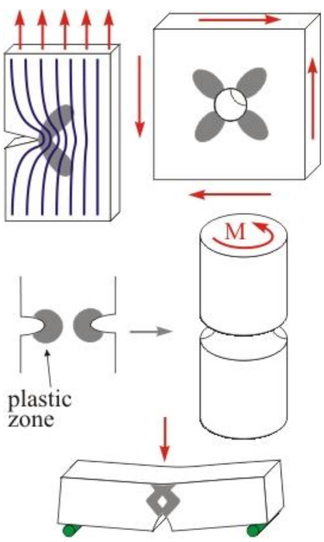


A stress concentration will occur near a broken rigid fiber in a flexible matrix. Delaminating fibers from the matrix will help "smooth" the redistribution of force lines.

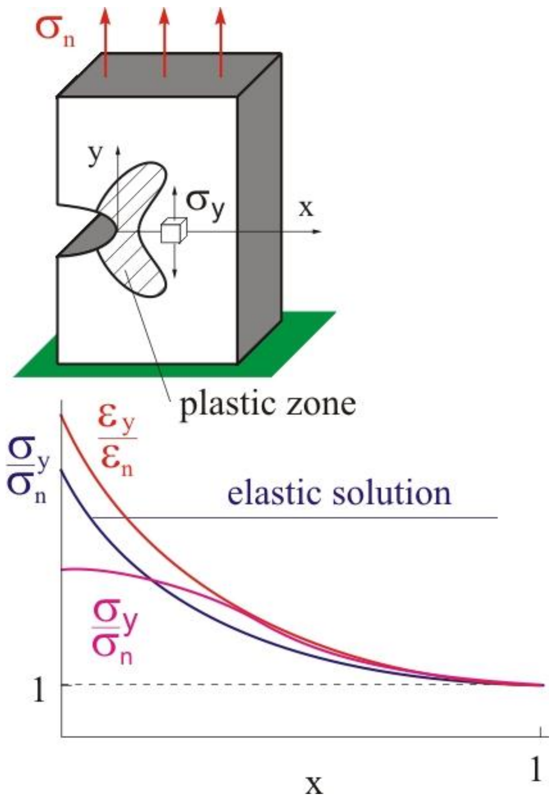


1.7 ELASTIC–PLASTIC STRESS CONCENTRATION

The nature of plasticity in metals is nonlinear shear deformation. The plastic region is different for tension, bending, pure shear, and other loading schemes. It forms an "ear" for tension, a round shape for torsion, and a plastic "hinge" for three-point bending. The plastic region allows for a decrease in stress concentration and for the redistribution of inner forces into neighboring areas. Unfortunately, strains increase in the plastic zone in comparison with in the elastic region. Strains are also finite; when the limit is reached, failure will occur. Force lines extend far from the notch tip in the elastic–plastic body.



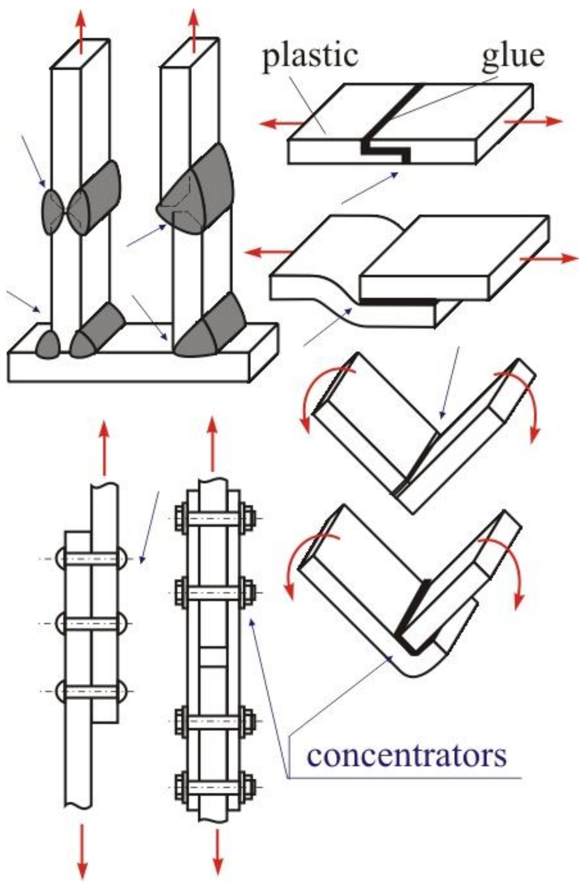
Under plastic deformation, the ratio of maximum stress to nominal stress decreases in comparison with the elastic case.



The opposite occurs with strains. A strain in the notch tip is larger than that in the elastic body. In real materials, stresses in the crack (the sharpest concentrator) will not reach infinity due to plastic or nonlinear deformation.

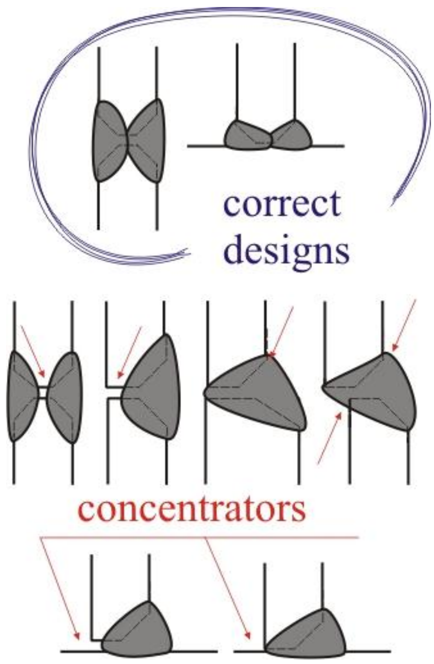
1.8 JOINTS: BOLTS AND WELDS

Joints are regions of stress concentration. Welds are "geometrical" stress concentrations that have inner residual stress (thermal stresses). There are zones where nondestructive testing can detect defects such as cracks (sharpest stress concentrator).

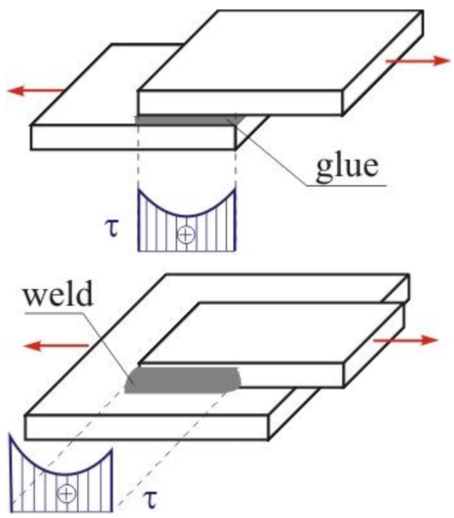


Glued joints also show stress concentration due to the non-uniformity of elastic properties and the change of geometry. Rivets and bolts are inserted into holes that are stress concentrators. Force lines must bend around the joints, which causes concentrations of the force lines at the joint.

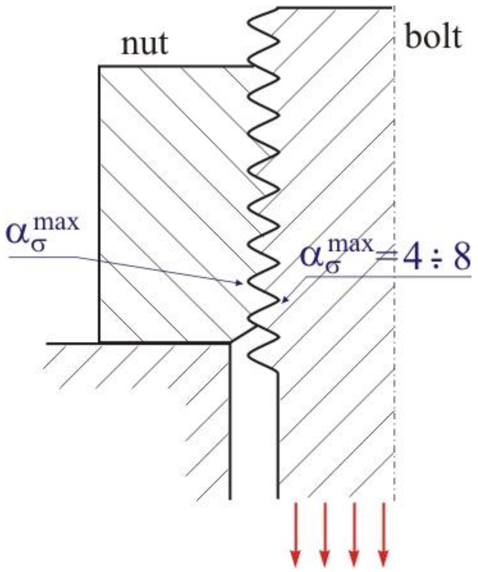
Designers should try to remove sharp corners and other flaws from weld joints. While this does not eliminate stress concentration, it can decrease the negative effect on the strength of the joints.



At the edges of glued surfaces and welds, there is an increase in shear stresses. The main reason for this increase is the difference in the rigidities of the connected plates at the ends.

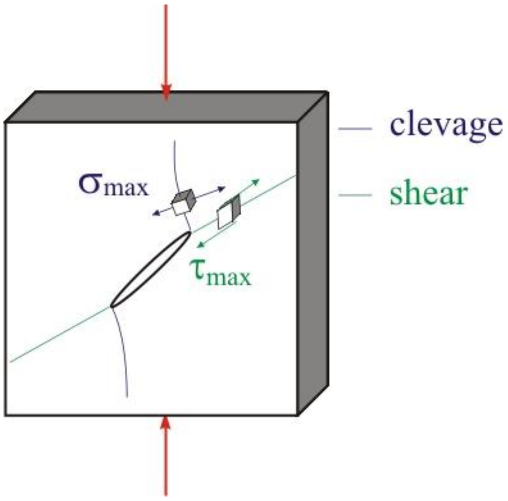


Thread is also a zone of stress concentration. There are stress concentrations in a bolt and a nut. The first thread has maximum loads and stress concentration. A smaller radius of curvature causes greater tensile stress. A less rigid bolt has a lower stress concentration in the first thread.

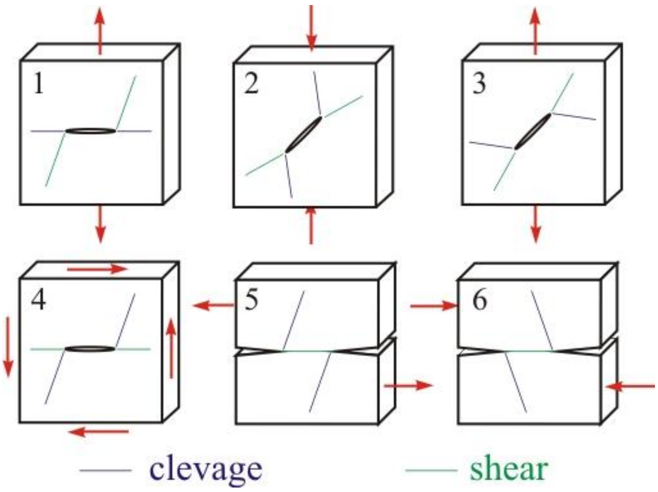


1.9 FRACTURE CRITERIA

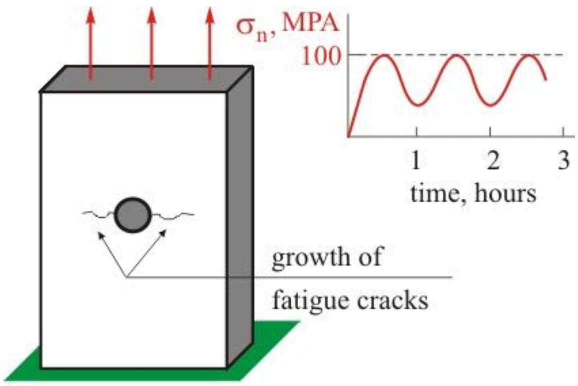
There are several types of fracture in a plate with a stress concentrator. If the plate material is brittle, the main mechanism of failure is cleavage, and new cracks will start perpendicular to the maximum tangential stress in the notch tip. For plastic material, a possible mechanism of failure is shear along the maximal tangential shear stress. Directions are not the same, as they depend on the loading scheme and geometry of the structure.

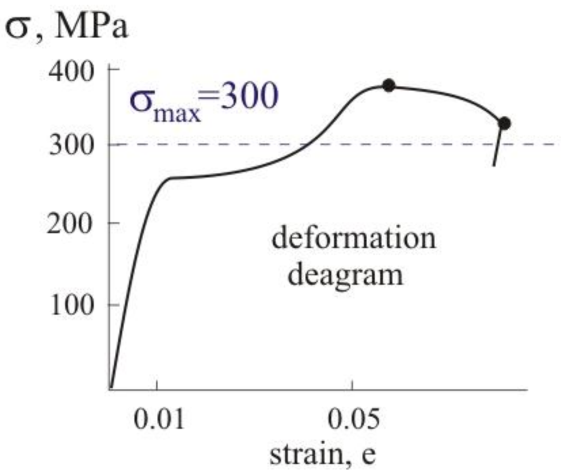


For tension (see items 1 and 3), the new cleavage is perpendicular to the applied force. Shear surfaces are at an angle to the force. For compression, the cleavage is parallel to the force and shear surfaces are again inclined. Under pure shear (4), the shear surfaces lie along the maximum axis of the elliptical hole and the cleavage is inclined. There are differences in the cleavage path for items 5 and 6.

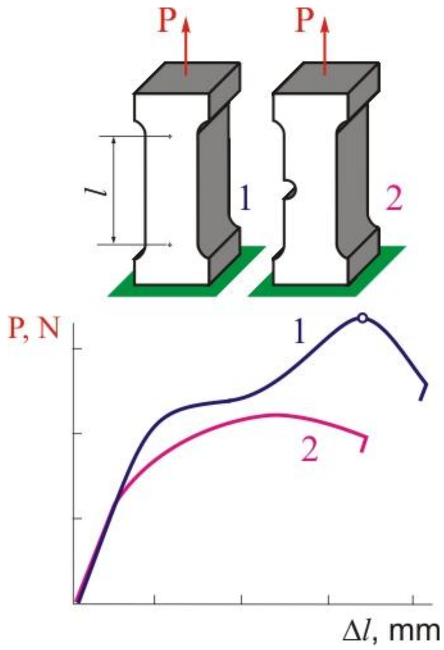


The main mechanism of fatigue crack initiation is local plastic deformation (e.g., on the neighborhood of a stress concentrator). The growth of fatigue cracks is higher if the maximum stress is greater than the limit of elasticity for the material. In this case, the time for crack initiation and growth is short.





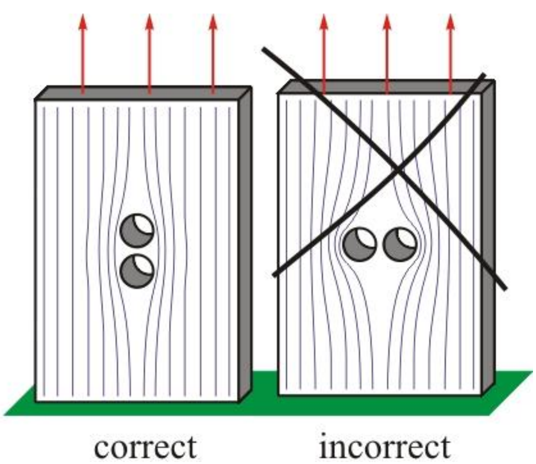
The comparison of these diagrams for two specimens shows that the notched specimen has less elongation (strain) and lower maximum load (stress). For plastic materials, there is no direct proportional relation to the theoretical stress concentration factor.



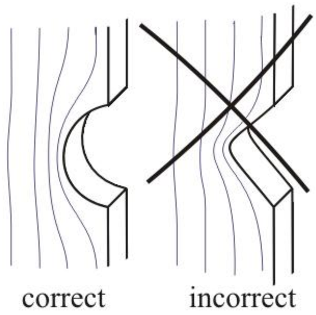
1.10 RATIONAL DESIGN

We need to consider many factors including stress concentration in order to determine design validity, cause of failure, and so on. Please remember what will generates less stress concentration:

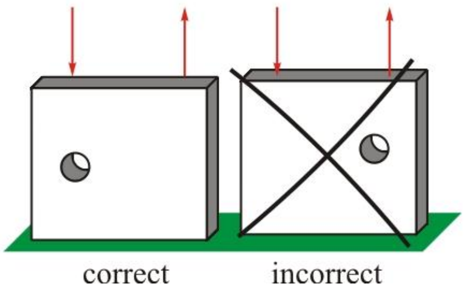
1) Shrinking obstacles on the force line path,



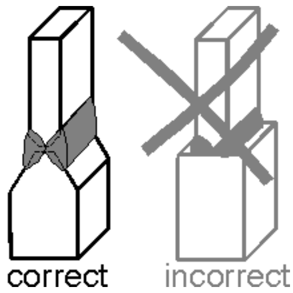
2) Enlarging the radius of curvature in a notch,



3) Removing stress concentrators from regions of tensile stress,



4) "Smoothing" the transition of rigidities, and



5) Creating greater distances between stress concentrators.

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