2. FRACTURE MECHANICS

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2.1 DEFECTS

Structural materials have inner defects such as cracks, which are extreme stress concentrators.

There are technological defects shown in diagrams A and B below, which are cracks that grew under exploitation into fatigue cracks, shown as diagrams C, D, F, corrosion attack (E), or thermal impact cracking. Improper exploitation such as scratching produces such defects. Leaking occurs through the cracks, allowing detection of defects before catastrophic failure occurs in pressure vessels.
The most dangerous defects are perpendicular to tensile stress in the material.

It is possible to approximate the curved front of the defect by an ellipse or circle. A three-dimensional singular defect can be replaced by projecting onto a surface perpendicular to the tensile stress. A surface defect is more dangerous than an inner defect of the same size. It is easier to inspect surface defects than inner pores by nondestructive control.
2.2 STRESS IN THE CRACK TIP

A crack is an obstacle in the path of force lines. A concentration of force lines affects the stress pattern in the cross section. According to the solution of the theory of elasticity for an extreme concentrator such as a crack, the maximum stress tends towards infinity. The solutions were obtained for ideal elastic material. Fortunately, the structural materials are not ideally elastic, as there are plastic deformations and microstructural changes in the crack tip. In Section 2.4 (Plasticity) we discuss the real value of stress. The other figure shows force line distribution and stress patterns for specimens with a crack under pure bending.

The figure below shows the original and deformed state (displacements are magnified) of a plate with a central crack. According to the theory, a sharp crack will transform into an elliptical hole with a shorter length and very small height.

There is a three-component stress field in the crack tip. The plot shows the stress distribution in the center of the specimen with a central crack, only the right part is shown.

In the crack tip there is a small zone where the formula for stress can be simplified. The magnitude $r_s$ is the size of the singular zone. Expression 2 is a singular solution. The expression is valid for all types of crack in tension with different geometries. The curve lies higher for larger nominal stress or for a larger crack length, but its form is the same. The size of singular zone is different for different schemes. In many cases, it is about 0.1 of crack length.
2.3 STRESS INTENSITY FACTOR

The solution to the theory of elasticity shows that two plates with different stresses and crack lengths can have the same stress distribution in the crack tips (in the singular zones). There are other combinations (for example «nominal stress-crack length») that also give the same stress distribution and the same stress intensity in the crack tip.

\[ \sigma_n = \sigma_n \sqrt{\frac{a}{x_1 - a}} \]  
\[ \sigma_y = \frac{\sigma_n}{\sqrt{2\pi r}} \frac{\pi a}{2r} \]

\[ K_I = \sigma \sqrt{\pi a} = \text{const} \]
Stress intensity factor (SIF) is a measure of stress intensity in the crack tip. A higher SIF means larger stress pattern at the line of crack continuation (axis X). Expression 3 is valid for a wide plate with a central crack.

For a plate where the crack size and width have the same order there is an additional coefficient $F$ for equation 3.

For small side cracks the correction factor is $F = 1.12$.

For small semi-elliptical cracks the maximum value of stress intensity is in point A and the correction factor is approximately equal to 1.12 for wide defects. The correction factor decreases with the larger ratio $a/B$ and the lower value of $2c/B$. 
The correction functions for other loading schemes can be found in the SIF handbooks. The value of $K_I$ allows us to determine the stress values at a distance $r$ in the crack tip. SIF $K_I$ is measured in MPa m$^{1/2}$ or in ksi in$^{1/2}$. 1 ksi in$^{1/2}$ is approximately equal to 1.1 MPa m$^{1/2}$. SIF $K_I$ is the magnitude of the ideal crack tip stress field.

\[
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} = \frac{\sigma_n\sqrt{\pi a}}{\sqrt{2\pi r}} \tag{6}
\]
2.4 PLASTICITY

Plastic deformation is unrecovered shear in zones of high stress concentration. The figure shows examples of plastic shear in the crack tip.

The stress in the crack tip of elastic-plastic models is finite. The maximum value is not as important as the magnitude of SIF.
For thick plates, the size of the plastic zone at the surface is bigger than in the center due to a three-dimensional state of stress in the central part of the plate. Despite this, stress intensity is higher in the center of a thick plate with a crack. There are different models of plastic zone in the crack tip that allow substitution of the plastic crack with a larger pure elastic crack. The size $r_y$ is determined by yield strength $s_{YS}$.

\[ r_y = \left( \frac{\sigma_y}{s_{YS}} \right) a \] (7)

2.5 FRACTURE TOUGHNESS

If stress is increased in a structure with cracks, crack initiation and fast growth will occur. The magnitude of the SIF $K_i$ at the time of crack extension is the maximum acceptable (critical) value for the material - $K_{IC}$. As nominal stress increases, the SIFs in the crack tips A, B, and C increase proportionally. $K_i(A)$ reaches its critical value $K_{IC}$ first. The structure fails at point A. Fracture toughness is a measure of resistance to the extension of a crack. $K_{IC}$ crack extension resistance has units of SIF [MPa m$^{1/2}$] or [ksi in$^{1/2}$].

Expression 8 above shows a condition of safe exploitation of a structure with cracks. The figure below shows that there are combinations of «nominal stress and crack size» that end in failure over the curve. There are combinations that do not end in failure. The nominal
stress cannot be larger than ultimate tensile strength, since this is a simplified correction for short cracks. Some plastic materials demonstrate slow crack extension before catastrophic failure. The safety margin in the presence of cracks is equal to the ratio of $K_{IC}$ to the maximum SIF.

![Diagram of stress vs. crack size](image)

The value of crack extension resistance is obtained by conducting a special fracture toughness test on standard fatigue pre-cracked specimens.

### 2.6 TEMPERATURE AND DEFORMATION RATE

There are mainly two kinds of fracture: brittle cleavage and ductile fracture. They demonstrate different values of the crack resistance. A material can have both types if the
test temperature varies widely. A decrease in temperature causes dramatic decrease of crack resistance at the transition temperature.

A crack progressing at high speed has no time to realize ductile fracture mode. An initiated crack will release stored energy of deformation and accelerate itself. If the speed reaches 1.5 km/sec, 4 seconds is enough to destroy 5 km of gas pipeline. Dynamic crack resistance $K_{ID}$ is lower than the static equivalent $K_{IC}$.

Corrosion cracking tests are conducted on double beam specimens with a wedging bolt. Corrosion causes the material in the crack tip to be brittle. As the crack begins and grows, the force and SIF decreases with increased flexibility of the beams.

The curve $K_{IC}$ constantly corresponds to the beginning of slow crack growth. When the release energy is enough to accelerate the crack, the dynamic crack extension begins.
2.7 SCALE EFFECT

An increase in sheet thickness is reflected in the size of the plastic zone and fracture mode. Thin plate usually fractures by shear, while the cleavage is typical for thick plates. The minimum value $K_{IC}$ is considered characteristic of the material.
Crack resistance also decreases if the size of the specimen increases.

Usually materials with smaller grain size demonstrate better mechanical characteristics.

The maximum stress in a plate with a crack cannot exceed the ultimate tensile strength, there is a correction function \( F_U \) for the SIF that accounts for this. The middle of the diagram is controlled by SIF without the correction functions.
2.8 MIXED MODE LOADING

There are three basic modes of crack (surface) displacement:
I - opening mode;
II - plane shear;
III - anti-plane shear;

There is also the combination of the three basic modes, known as mixed mode.

There are a few modes of failure, for example by cleavage A or by shear B. There are practical cases when the fracture mode is cleavage under pure torsion or plastic shear under nominal tensile stress. The crack extension A is perpendicular to maximal tangential stress.
The initial direction of crack extension depends on the loading scheme and type of the material. Brittle materials usually fracture by cleavage (A). Plastic shear is typical for ductile materials and specimens with a narrow cross section.

Cracks perpendicular to the maximum tensile stress have larger stress intensity. Crack A, inclined to the stress is equivalent to crack B if they have the same projection. They start at the same stress.

Shear stress distribution is similar to the normal stress pattern.
2.9 FAILURE PREVENTION

The most reliable approach is "No cracks - no problems," but it is not easy to accomplish. Engineers try to have high-strength, high ductility, high crack resistance, and faultless structures.

When all is not possible, there are some methods of prevention of catastrophic failure, as shown below: increase crack resistance by ductile material (A - C), by local heating (D); decrease SIF by placing holes on the crack path (E, F), by patching (G), by stiffing elements (H) or by using composite materials (I).

"Leak-before-break" is an effective strategy to prevent catastrophic failure of pressure vessels (J). It is better to allow a semi-elliptical crack to grow through the wall (J) and to detect it by leaking than to have the dynamic start and failure of the whole vessel. There are two characteristics of the material: crack resistance for semi-elliptical crack $K_{ICT}$ and crack resistance for through crack $K_{IC}$.

Patching accepts inner loads from the cracked plate and decreases SIF. If the connection is strong enough the SIF does not exceed a certain value.
Stringers on the crack path change the critical combination of "stress - crack size." The reinforced structure has a larger crack size at the same stress.

\[ \text{Stress } \sigma \cdot \text{MPa} \]
\[ \text{Crack size } a, \text{mm} \]

2.10 Failure analysis

Brittle and ductile fractures have different views of the broken surfaces. It is possible to reveal defects at the broken surfaces. Defects in the zones of tensile stress are usually the main source of fatigue cracks.

Crack resistance depends on crack orientation and is usually bigger for cracks perpendicular to the rolling direction (L-T).
Fracture toughness and ultimate tensile strength are used for short cracks in pressure vessels.

Fatigue crack rate depends on SIF. The larger the SIF, the faster the crack growth rate. The figure below shows a typical diagram of the "fatigue crack size - number of cycles."

Short cracks starting from a hole are in the zone of stress concentration (stress concentration factor is 3). Relatively long cracks passing through a hole are considered cracks augmented by the diameter of the hole.
REFERENCES


