6. STRUCTURAL SAFETY

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6.1 RELIABILITY

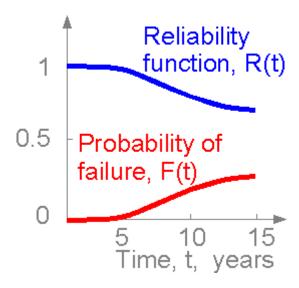
Dependability is the ability of a structure to maintain its working parameters in given ranges for a stated period of time. Dependability is a collective term used to describe the availability performance and its influencing factors: reliability performance, maintainability performance and maintenance-support performance. Dependability is used only for general descriptions in non-quantitative terms.

Maintainability is a measure of the ease and rapidity with which a system can be restored to operational status following a failure.

Reliability is the probability of component or system success under stated conditions for a stated period of time. The length of time to the moment when the structure fails is not exactly known. The time is a random variable. Reliability is characterized by the reliability function **R(t)**:

$$R(t) = P\{t > T\}$$

where \mathbf{P} is the probability that the operating time to failure \mathbf{t} exceeds the stated period of time \mathbf{T} . Probabilities are numbers between 0 and 1.



The reliability function is related to the probability of failure function F(t) and its density function f(t):

$$F(t) = 1 - R(t)$$
; $f(t) = d F(t) / dt = - d R(t) / dt$;

The domain of the cumulative distribution function $\mathbf{F(t)}$ is from zero to infinity, while its range is the interval $[0 \dots 1]$.

Failure rate $\lambda(\mathbf{t})$ is an important parameter of reliability. Every random variable has its associated cumulative distribution function $\mathbf{F}(\mathbf{t})$, which increases (accumulating probability) as t increases.

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = -\frac{d \ R(t)}{R(t) \cdot dt}$$

If the failure rate is constant (exponential law of reliability) the dependence between failure rate and reliability function can be simplified. Statistical estimators of the function can be obtained by the following expressions:

$$R(t) = 1 - \frac{n(t)}{N}, F(t) = \frac{n(t)}{N}$$

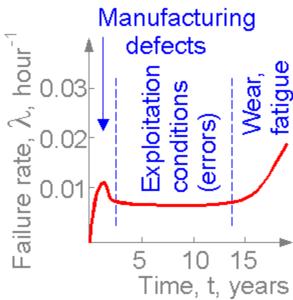
$$\lambda(t) = \frac{n(t+\Delta t) - n(t)}{N \cdot \Delta t} = \frac{1}{t_0}$$

where $\mathbf{n(t)}$ is the number of failed elements to time \mathbf{t} , \mathbf{N} is the total number of elements, and $\mathbf{t_o}$ is the mean time to failure.

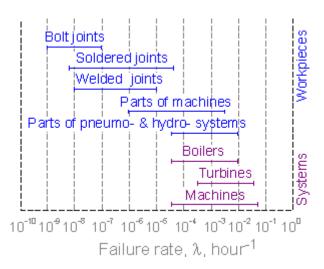
There is some standard behavior of the dependence curve of failure rate I versus time t:

- 1. failures due to manufacturing defects (cracks, inappropriate materials, poor design);
- 2. failures due to conditions of service (exploitation errors);
- 3. failures due to accumulated damage (wear and fatigue).





The figure shows typical ranges of failure rate for different engineering systems. The parameter changes over a wide range.



6.2 DISTRIBUTION FUNCTIONS

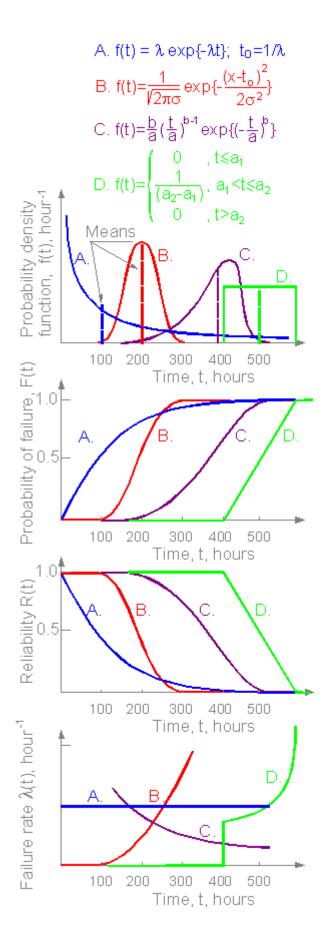
The probability of failure functions (cumulative distribution functions) F(x) and density functions f(x) are profiles that contain all the relevant information about the statistical properties of a random variable. The most useful information about distribution is its location (the mean) and its dispersion (the variance). There are four well-known distribution functions to describe reliability:

- A. Exponential
- B. Normal (Gaussian)
- C. Weibull
- D. Uniform

For structures consisting of a large number of structural elements with independent failures, the reliability function is exponential. The Weibull law is used if failures are connected with damage accumulation (wear-out, fatigue). The uniform law is used if the only information available about the distribution is lower and upper boundaries. The normal distribution is the most frequently used and is symmetrical about the mean value. At this mean the probability of failure and reliability are equal:

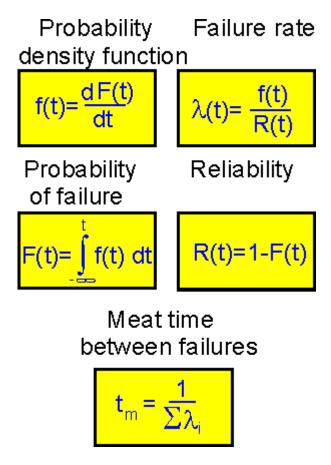
$$F(t) = R(t) = 0.5$$

The summed area under the probability density function is constant and always equal to 1. If the summation is made at segment $[0,\mathbf{t}]$, then the sum is equal to the probability of failure $\mathbf{F}(\mathbf{t})$ to time \mathbf{t} . The (failure) probability function $\mathbf{F}(\mathbf{t})$ is equal to zero at the left end or 1 at the right end of the axis (infinity).

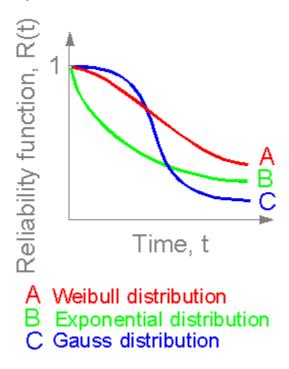


The reliability function **R(t)** characterizes the probability of structural integrity. All distributions can be characterized by the mean and the dispersion or the standard deviation.

There are special mathematical formulas to calculate the parameters. For a small number of structural elements (for example 10), the reliability function of the whole structure corresponds to the distribution law of its elements. If elements are described by normal law, the structure has also normal distribution. Mean time between failures $\mathbf{t}_{\mathbf{m}}$ is inversely proportional to the summed failure rate of a structure.

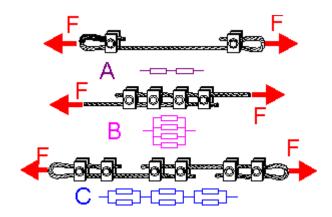


Sudden failures correspond to sharp changes in the reliability function. Sudden failures can be described by Gaussian (normal) distribution.

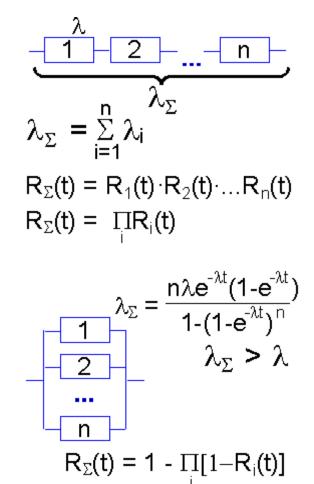


6.3 STRUCTURAL MODELS OF RELIABILITY

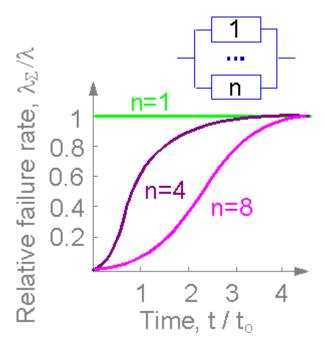
Machines and engineering structures consist of a large number of structural elements that can fail. The way elements are connected and their reliability define the reliability of the whole structure. There are examples with series connection A, parallel connection B and a combination C.



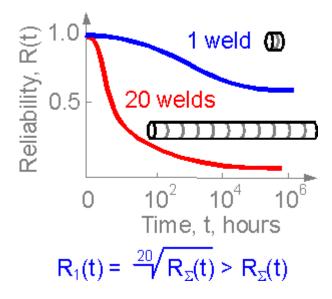
For a series system, failure of an element results in failure of the whole structure. Reliability of the structure RS is the product of the reliabilities of its elements. The failure rate of the series structure is the sum of failure rates of its elements. For a parallel system, the failure rate is smaller than the failure rate of its elements.



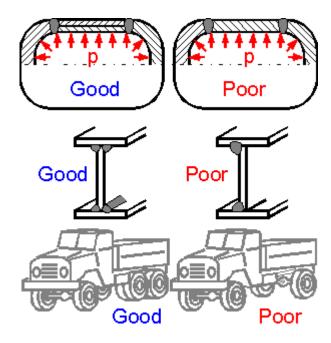
The failure rate decreases if the number of redundant elements increases. The failure rate also decreases over time, except in the initial and final stages.



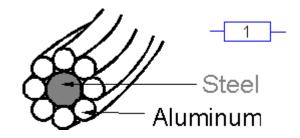
An example of an inverse problem is that if the reliability of the entire oil pipeline is known, then the reliability of a single butt-weld can be predicted.



The figure shows examples of structures with parallel connections. The analysis shows that the reliability of structures with a larger number of redundant elements is higher. This does not mean that parallel connection is always economically effective.

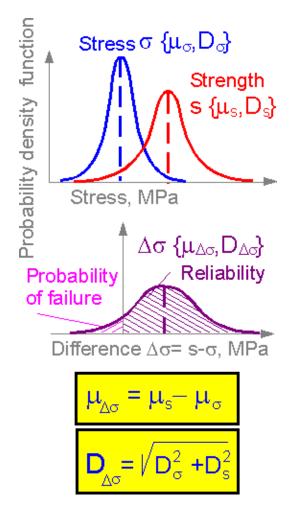


Among structural elements, there are parts that do not carry the load and the failure of these parts has no significant effect on the load-carrying ability of the structure. For example, in a 7200 volt electric power line ACSR (aluminum conductor, steel reinforced), the aluminum wire does not accept enough of the load. The failure of the steel wire results in failure of the line. A single element (steel wire) reliability model can be proposed to judge the carrying ability of the electric line.

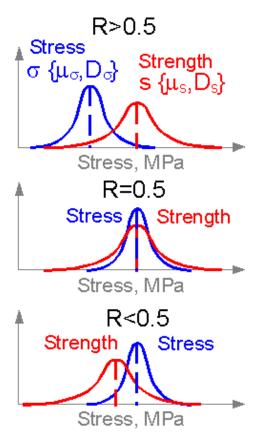


6.4 LIMITING STATE

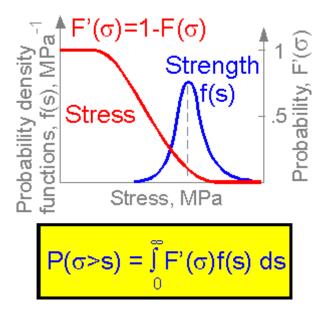
If the probability density functions for strength and stress of a structure are described by normal functions with known means mi and standard deviation $D_{i,}$ the probability density function for difference in strength and stress is also a normal distribution.



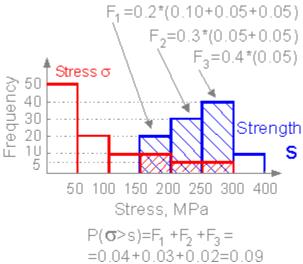
The reliability is equal to 0.5 if the means of the distributions coincide. It is larger than 0.5 if the mean for stress is smaller than for strength.



The summed area under a density function is equal to 1. The probability of failure is equal to the integral over the range where both functions f(s) and F'(s) are non-zero.

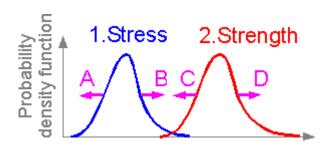


The summation is over three intervals. The probability of the failure of a structure is 0.09 and its reliability is 0.91.

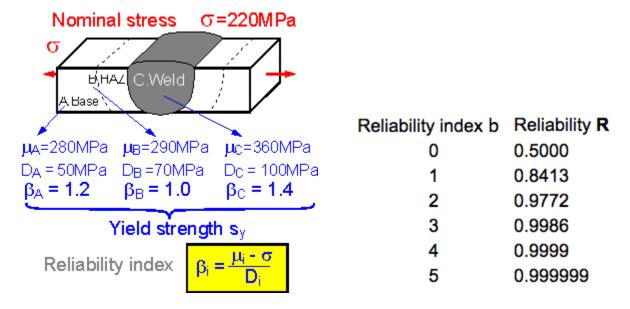


Engineering changes can alter the positions of the probability density functions.

- A. Cross section area increase results in a stress decrease.
- B. Stress concentration results in a stress increase.
- C. Service in low temperatures results in brittle fracture (strength decrease).
- D. Heat treatment of welded joints results in residual stress decrease (strength increase).



The reliability index is a measure of the reliability of a structure. The minimum reliability index of structural components corresponds to the reliability index of the whole structure. The smaller the reliability index bi, the more probable the failure.



6.5 STRUCTURAL FRACTURE MECHANICS

Structural fracture mechanics is the field of structural engineering concerned with the study of load-carrying structures that includes one or several failed or damaged components. It uses methods of analytical solid mechanics, structural engineering, safety engineering, probability theory, and catastrophe theory to calculate the load and stress in the structural components and analyze the safety of a damaged structure.

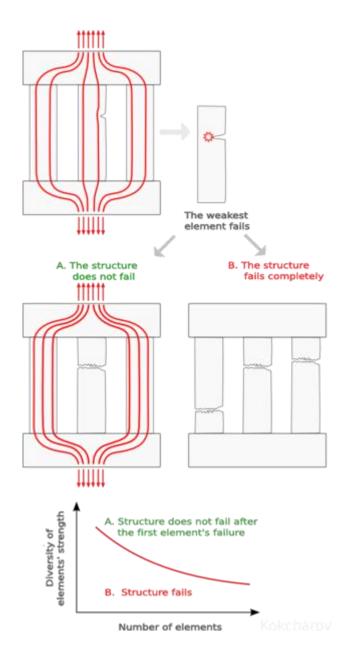
Analogy Between Fracture Mechanics of Solid and Structural Fracture Mechanics

	Fracture Mechanics	Structural Fracture Mechanics
Model	Solid with a crack	Multi-component structure with a failed component
Defect driving force	Stress intensity factor	Overload stress
System property	Fracture toughness	Reserve ability / Structural robustness

There are different causes of the first component failure: 1) mechanical overload, fatigue (material), unpredicted scenario, and so on; 2) "human intervention," such as unprofessional behavior or a terrorist attack.

There are two typical scenarios:

- A. A localized failure does NOT cause immediate collapse of the entire structure.
- B. The entire structure fails immediately after one of its components fails.



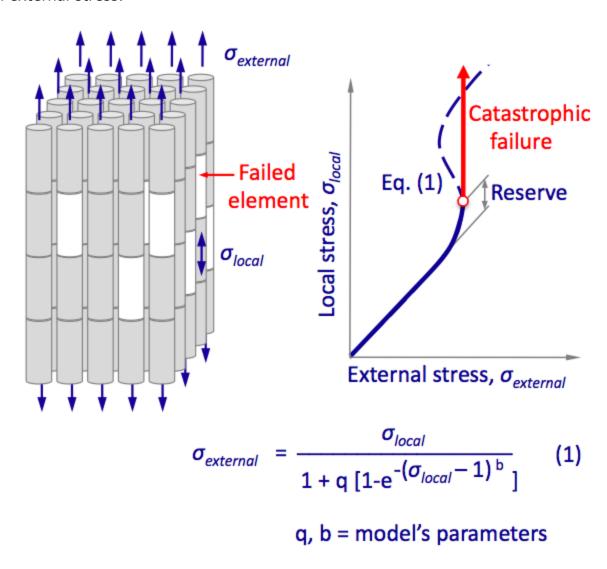
If the structure does not collapse immediately, there is a limited period of time until the catastrophic structural failure of the entire structure occurs. There are a critical number of structural elements that define whether the system has reserve ability or not.

Safety engineers use the failure of the first component as an indicator and try to intervene during the given period of time to avoid the catastrophe of the failure of the entire structure.

For example, "Leak-Before-Break" methodology means that a leak will be discovered prior to a catastrophic failure of the entire piping system occurring in service. It has been applied to pressure vessels, nuclear piping, and gas and oil pipelines, to name but a few examples.

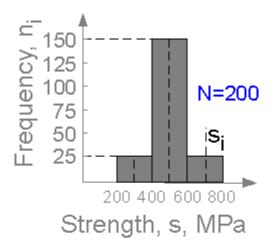
The methods of Structural Fracture Mechanics are used as checking calculations to estimate sensitivity of a structure to its component failure.

The failure of a complex system with parallel redundancy can be estimated based on probabilistic properties of the system's elements. The model supposes that failure of several elements causes the overloading of neighboring elements. The model equation (1) shows the relationship between local and external stresses. The equation (1) is similar to the cusp catastrophe behavior. The theory predicts reserve ability of the complex system and the critical external stress.



6.6 DISPERSION

The histogram shows how many specimens have strength in the corresponding intervals. The experimental data and corresponding analytical distributions can be characterized by mean value μ_s and standard deviation \mathbf{D}_s . The standard deviation is expressed in terms of the same units of measurement as the variable μ .



$$\mu_s = \frac{1}{N} \sum_{n_i s_i} Mean$$

$$\mu_s = \frac{1}{200} (25.300 + 150.500 + 25.700) = 500MPa$$
Standard deviation

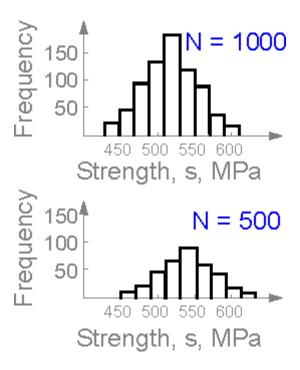
$$D_s = \sqrt{\frac{1}{N} \sum_{i} n_i (s_i - \mu_s)^2}$$

$$D_{S} = \sqrt{\frac{50 \cdot 200^{2}}{199}} = 100 \text{ Mpa}$$

Variation coefficient

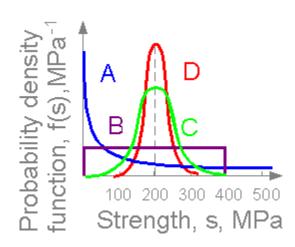
$$V_{\rm S} = \frac{D_{\rm S}}{\mu_{\rm S}}$$
 $V_{\rm S} = \frac{100}{500} = 0.2$

The standard deviation does not depend on the number of tests N if the number is large. The shapes of the two histograms are similar. The standard deviations are approximately equal for both examples.



All distributions (exponential A, uniform B, normal C and D) correspond to the mean μ_s =200 MPa. The areas under the curves are equal. The larger part of the area is placed far from the mean, the bigger standard deviation \mathbf{D}_s :

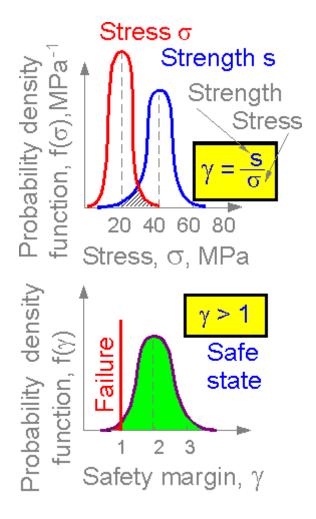
$$D_s(A) > D_s(B) > D_s(C) > D_s(D)$$



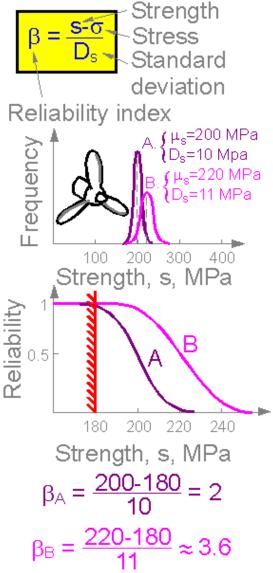
The mean of the safety margin is equal to the ratio of the means for strength and stress: μ_{γ} = 40 / 20 = 2.0

There is probability of failure for the structure:

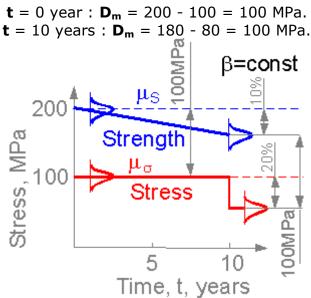
$$P(\gamma) > 0 \text{ if } \gamma < 1$$



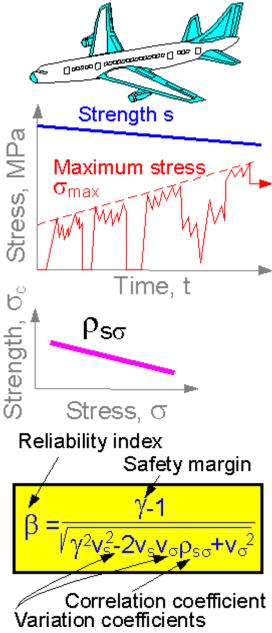
The reliability index helps to compare two variants. If the acting stress is constant ($\sigma = 180$ MPa), the reliability index depends on variation parameters of strength. The bigger the reliability index, the bigger the reliability of the structure.



As a result of damaged growth in a heat exchanger, the strength of the material decreases by 10%. To keep the initial reliability, the engineers decrease the stress due to vapor pressure by 20% after 10 years of service. In this case, the reliability index is equal for both time \mathbf{t} =0 and \mathbf{t} =10 years.

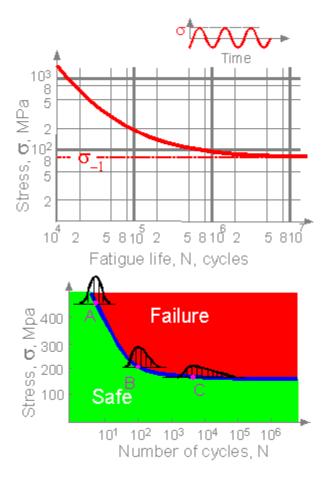


A carbon-fiber reinforced plastic element of an airplane has micro-damages, and its rigidity and strength decrease with time. The maximum stress increases due to smaller rigidity of the elements. There is a correlation between stress and strength, and the correlation coefficient affects the reliability index of the structural elements. The negative correlation coefficient decreases the reliability index, whereas the positive correlation coefficient between stress and strength is preferential.

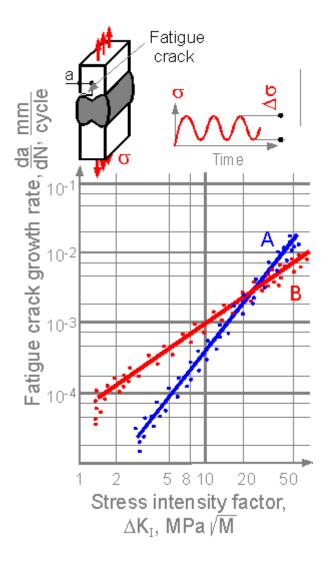


6.7 DURABILTY

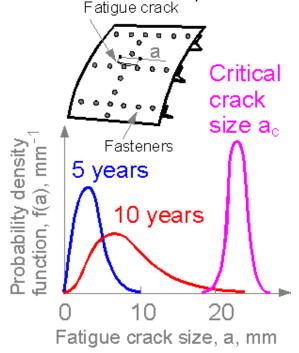
The durability of structures is connected with degradation processes such as fatigue, creep and others. The "Stress - Number of cycles to failure" (**S-N**) diagram describes the fatigue in metals. Fatigue strength s_{-1} corresponds to the horizontal segment of the curve. If there is no horizontal segment then the value can be assumed at $\mathbf{N} = 10^6$ or $\mathbf{N} = 10^7$. At high stress the fatigue life and its dispersion are small, whereas at low stress the probability density function is not symmetrical and its dispersion is bigger.



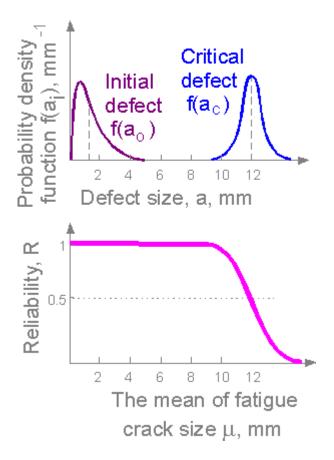
The initiation and growth of fatigue cracks occur under cyclic loading. The figure shows an experimental diagram – "fatigue crack growth rate vs. stress intensity factor" – for two buttwelds made from different steels. The smaller the fatigue crack growth rate, the better. Furthermore, the fatigue macro-crack growth is longer at the initial stage if the values of stress intensity factor, A, is small.



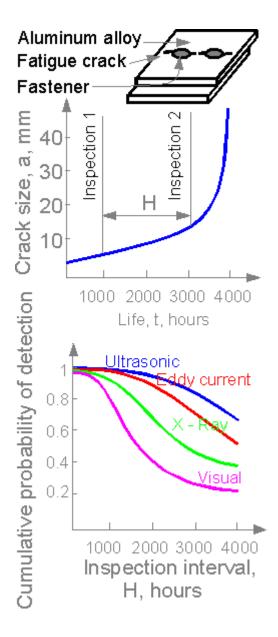
Over time, the dispersion increases for fatigue cracks. Short cracks have no significant increase, but the longer cracks grow faster. Theoretical distributions for fatigue crack go from zero to infinity, meaning that the theoretical reliability measure cannot be equal to 1.0.



A fatigue crack grows in a structural element from initial size a_{\circ} to its critical size a_{c} . The reliability is approximately equal to 1 for a small crack. The fatigue crack reaches its critical size at failure and the reliability decreases. When the means coincide, the reliability is equal to 0.5.



Airplane fuselages are periodically inspected for fatigue crack identification. Probability of crack detection depends on the method of nondestructive testing (NDT) and the inspection interval. The probability of crack detection is approximately equal to 1 for short intervals and decreases as the intervals increase. The ultrasonic NDT is the most effective technique.

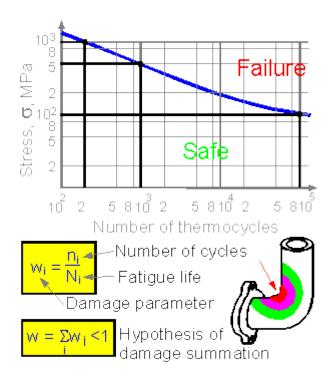


The figure below shows the dependence of maximum stress on the number of cycles sustained by an element of a pressure vessel. The engineers can plan the service at different stress, for example:

1000 MPa - 100 cycles 500 MPa - 500 cycles 100 MPa - 50000 cycles

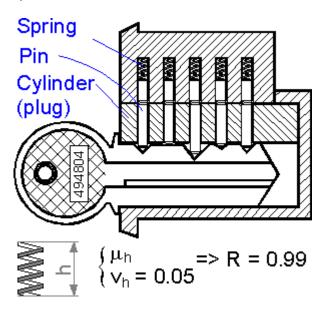
The element fails if the sum damage parameter exceeds 1. In our case, the element fails because:

$$\mathbf{w} = 100/200 + 500/1000 + 50000/100000 = 0.5 + 0.5 + 0.5 = 1.5 > 1.0$$

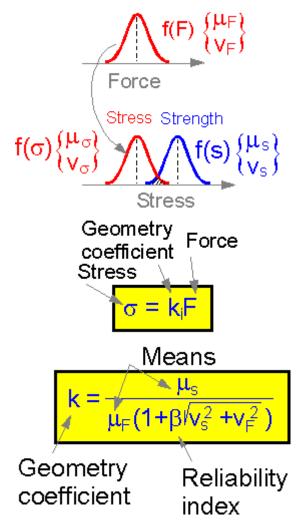


6.8 DESIGN BY RELIABILITY CRITERION

Geometrical characteristics of a machine's parts can be selected by reliability criterion. The parameters (thickness, length, cross-sectional area) can be chosen so that the total reliability of a machine is equal to the required value. If the height dimension h of a spring corresponds to the required reliability, the reliability of the safety lock consisting of five springs is smaller. In this example, the geometrical parameter is the height. This means that the variation coefficient defines the reliability of a "spring-pin" set and, together with other geometric characteristics, the reliability of the whole lock.

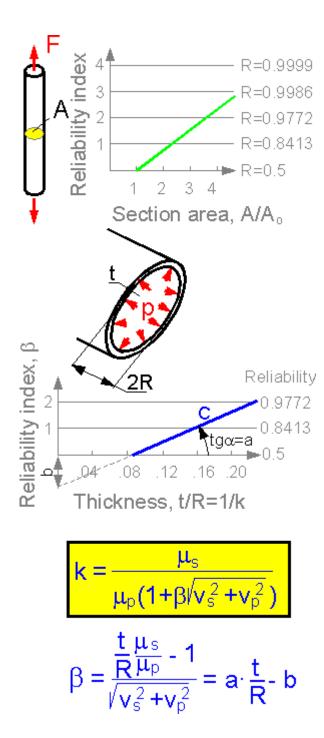


The condition of failure is defined at the stress coordinate axis. To transform the known statistical data for force at the axis, a special geometrical coefficient $\mathbf{k_i}$ is used. Variation coefficients for force and stress are equal if the cross-sectional area is constant. The last formula relates the geometry coefficient and the reliability index.

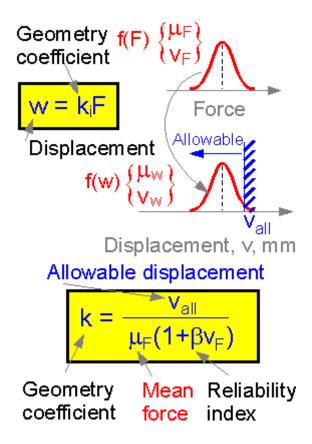


There is linear dependence between the cross-sectional area A and the reliability index for the bar if all other variables are constant.

The dependence "reliability index - thickness" is also linear for the pipe.



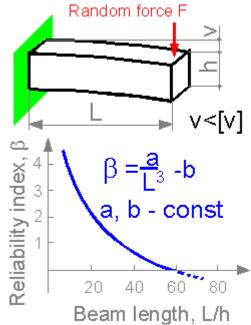
A restriction can be specified for maximum deflection, linear displacement or angle of twist. For such a restriction, the dependence between the geometrical coefficient and the reliability index is also linear. There are one-sided (A and B) and two-sided (C) restrictions, the latter of which is more severe.



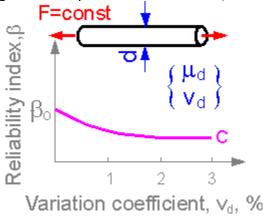
The table shows values of geometrical coefficients for different loading conditions. The geometrical coefficient is the ratio of Stress / Force for tension if the strength criterion is used.

Geometry coefficient k					
Scheme	Strength	Rigidity			
F A E - modulus of elasticity	$k = \frac{1}{A}$	k = <u>1</u> EA			
$I = \frac{bh^3}{12}$	$k = \frac{L \cdot h}{2I}$	$k = \frac{L^3}{3EI}$			
J= $\frac{\pi r^4}{2}$ G= $\frac{E}{2(1+\mu)}$	$k = \frac{\tau}{T}$ $k = \frac{r}{J}$	$k = \frac{\theta}{T}$ $k = \frac{L}{GJ}$			
t 2R	$k = \frac{\sigma}{p}$ $k = \frac{R}{t}$	$k = \frac{R^2}{Et}$			

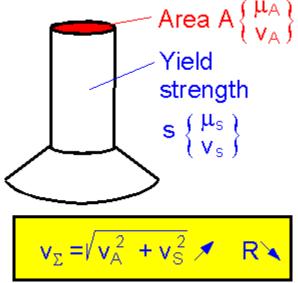
To meet a rigidity criterion for a cantilevered beam, the reliability index β is inversely proportional to L_3 . The reliability for the rigidity criterion decreases with the increase of beam length.



Dispersion in the size of wire (the variation coefficient) is reflected in dispersion of inner stress or inner force. The bigger the dispersion of the force, the smaller the reliability index b.



The yield strength and cross-sectional area are independent variables. The bigger the dispersion, the smaller the reliability of the rivet.



6.9 RISK

A dictionary definition of risk is "to expose to the chance of injury or loss." For technical systems, risk is a measure of probability of failures or accidents during a given period of time and/or measure of their consequences. The consequences can be social (severe injury or fatalities), technical (out of service), economical (financial losses) or ecological (environment contamination). The basic formula for risk is:

Risk = Probability of failure * Consequences

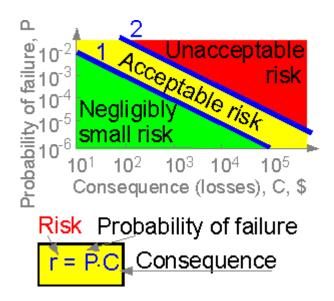
$$r = F * C$$

The risk is measured in units of consequence (losses), for example people and money. For a complex structure, a risk table is used.

The total risk for a system is the sum of the risks of its elements (failure scenarios). Engineers often use the frequency or failure rate, [year⁻¹] instead of probability of failure in the formula for risk assessment.

There are two levels of acceptable (1) and unacceptable (2) risks, and there are three ranges of risks: negligibly small, reasonable losses, and unacceptable losses. Acceptable risk of severe injury in Europe is 10^{-6} 1/year. The range of real risks is 10^{-4} to 10^{-10} per year⁻¹. The most dangerous objects are chemical plants, oil refineries and transportation systems.

A parameter such as risk is important in comparative analysis. Nuclear power plants are unique engineering structures, and there are few similarities between any two plants. The summed service time of all nuclear reactors in the world does not exceed 10^6 years. Unfortunately, there have been human losses at nuclear power plants.

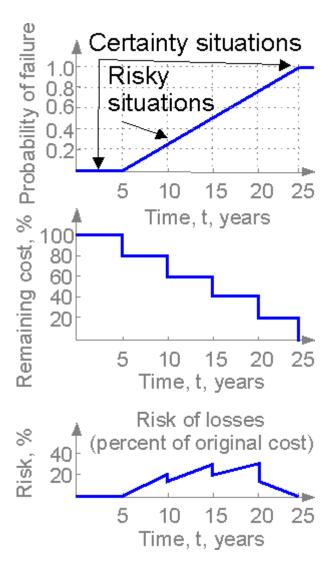


A probabilistic safety analysis of a nuclear power plant piping system can be made by corresponding classes of consequences.

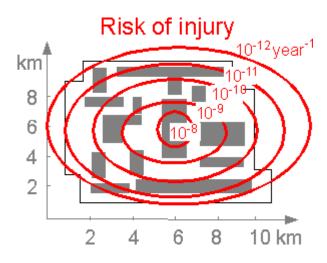
Initiating events	Frequency	Exposure	Number of available backup systems / trains			
			0	1	2	> 2
Partial loss of feed	Anticipated	Long				
water	events I >1 year ⁻¹	Short				
Loss of feed water,	Infrequent	Long				
loss of off-site power	events I = 0.1 - 0.01 year ⁻¹	Short				
Streamline break,	Accidents	Long				
significant leakage	l<0.01 year ⁻¹	Short				

Consequence category High
Consequence category Medium
Consequence category Low

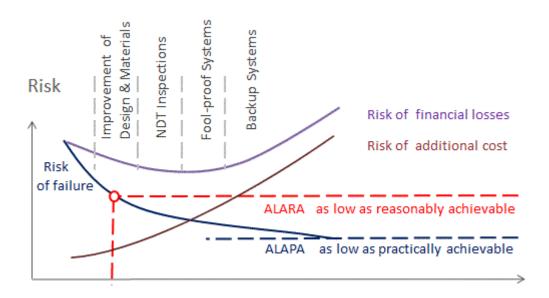
With time, the probability of failure and the remaining cost of a machine affects the risk measure. Situations where the probability is equal to 0 or 1 are certainty situations - not risky situations. Risk changes with time for the machine with shown parameters.



The figure below shows a map of a plant site with areas of risk of severe injury to a person per year. The information helps to identify the safest location for personnel.

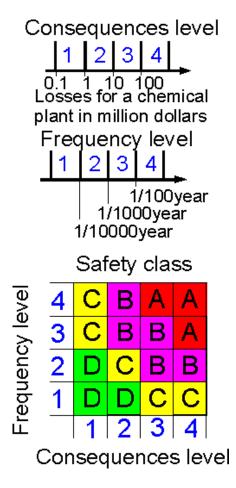


Risk of failure can be decreased to an ALARA (As Low as Reasonably Achievable) level by accounting for all economic aspects of a new plant. Risk of failure can be decreased to an ALAPA (As Low as Practically Achievable) level. For most complex technical systems, risk of financial losses due to structural element failure cannot be decreased to zero.

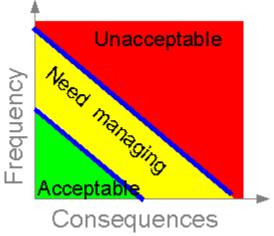


6.10 SAFETY CLASSES

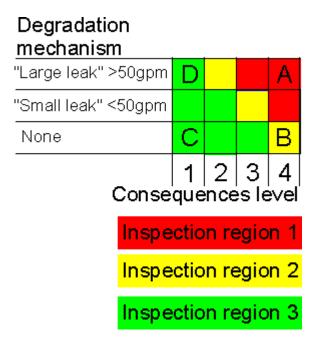
All failures can be categorized according to frequency of occurrence and consequence of failure. The figure shows the classification of consequence levels for chemical plants. Frequency and consequence levels can be described by a table of safety classes where A is the most dangerous state and D is the safest state.



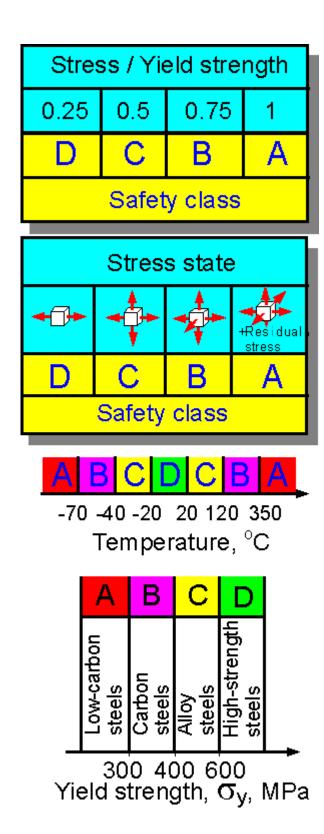
Besides levels of consequence and frequency, the numerical values of the parameters define zones of acceptable and unacceptable risks.



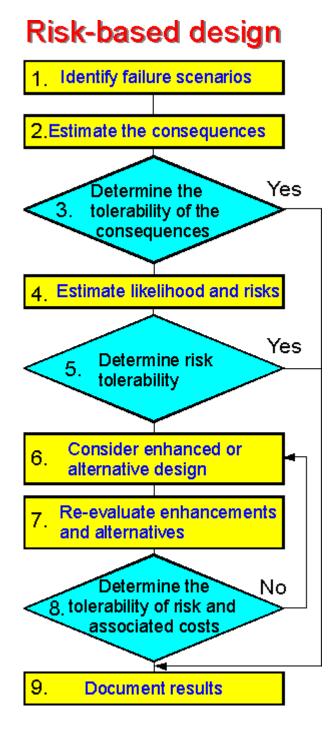
For complex engineering systems, nuclear power plant piping for example, the classification can be made by consequence and failure mechanism. This classification defines priority in the identification of inspection regions. The highest priority is given to large leakages in pipe, nozzles, and welds with severe consequence to the safety of the plant and personnel - A.



Safety classes can be categorized by stress state (from unidirectional to three-dimensional + residual stress), stress / yield strength ratio, materials or temperature. Safety class of a structure is the minimum rating after considering all the factors.



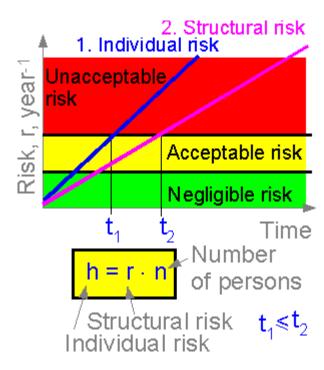
The risk-based design consists of nine steps, the first of which is the identification of failure scenarios.



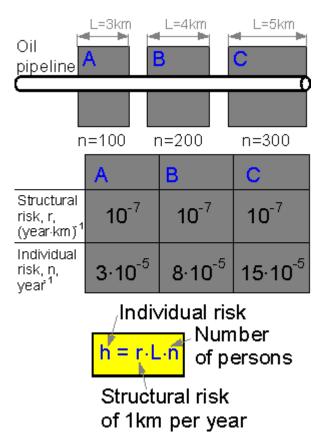
6.11 RISK: STRUCTURAL AND SOCIAL

Structural risk is connected with individual and social risks. Individual risk is equal to structural risk times the number of structural elements and number of people at risk.

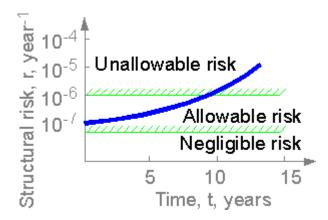
The figure below shows the structural and individual risks for a space shuttle. The individual risk of a crew member is the most important consideration, and service with unacceptable individual risk ($t > t_1$) cannot be recommended. The more crew members, the higher the individual risk.



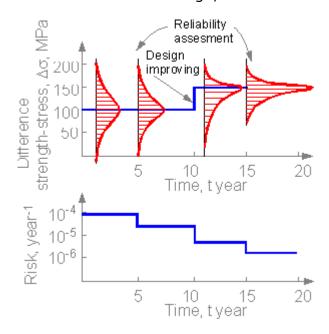
If an oil pipeline passes near a settlement, the structural risk for L kilometers of the pipeline is equal to L times the structural risk of one kilometer. The more people in the settlement, the bigger the individual risk.



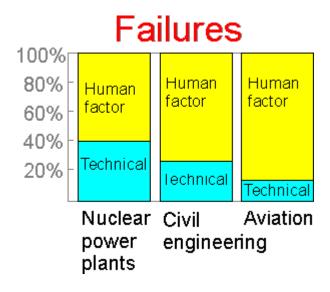
The structural risk of a severe accident of a structure can increase with time. If the risk exceeds the allowable value from corresponding industry standards, then the management of long-time service should be considered.



Safety assessments, knowledge of the likelihood of failure scenarios, and the design improvement can decrease risk. The more knowledge, the smaller the risk.



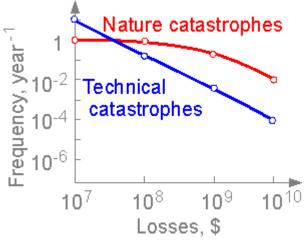
The human factor plays a significant role. Nuclear power stations have backup systems and special nondestructive testing and inspections that decrease risk.



Statistics can help choose a low-risk profession. Fortunately, other factors such as salary and interest also affect the decision.

System	Frequency of severe human injury			
Automobile accident	270 * 10 ⁻⁶ year ⁻¹			
Fire and explosion	40 * 10 ⁻⁶ year ⁻¹			
Machines and mechanisms ₁₀ * 10 ⁻⁶ year ⁻¹				
Aviation	7.5 * 10 ⁻⁶ year ⁻¹			
Electricity	5 * 10 ⁻⁶ year ⁻¹			

Losses due to natural catastrophes such as tornadoes, earthquakes, and forest fires at level 10^{10} are more critical. More losses are connected with natural catastrophes than technical natural catastrophes.



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